Modeling flow over superhydrophobic surfaces

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Introduction

Recently, much attention has focused on particular types of coating, aimed to reduce the skin friction drag in laminar or turbulent flows [1], with a variety of engineering applications. The working mechanism is based on the their super-hydrophobic behavior, which is well know in Nature, since the description of the *lotus effect* by Barthlott and Neinhuis [2].

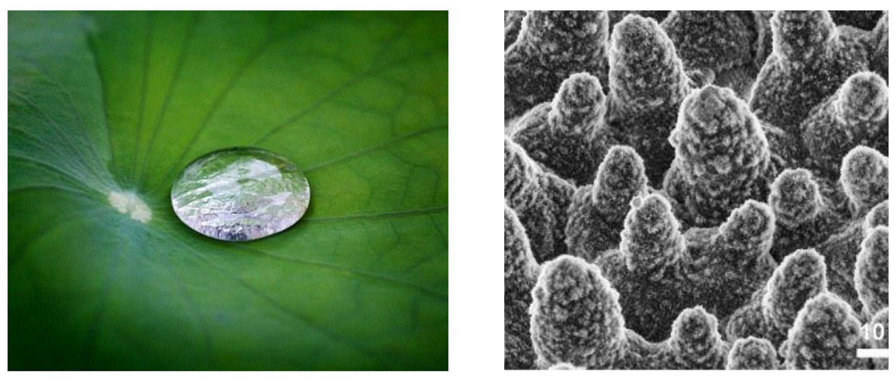

Figure 1: A lotus leaf showing a super-hydrophobic behavior, together with details of wall texture at microscope.

The key feature is the presence of a micro- or nano-structured surface that traps the air into small pockets over which the water can flow with low friction. The persistence of this condition, called Cassie-Baxter state, is as crucial as difficult to maintain in time since the gas layer can easily depleted. The aim of this research is to develop a computation model, based on a multiscale approach, in order to characterize the drag reduction induced by super-hydrophobic surfaces.

The multiscale approach

The study is carried out by solving:

- One microscopic problem, which describes the flow in the proximity of the wall protrusions. The goal is to compute the **protrusion heights** $\boldsymbol{\lambda}_{||}$ and $\boldsymbol{\lambda}_{\perp}$, which quantify the effectiveness of the wall structures in term of drag reduction.
- One macroscopic problem, which evaluates the effect of super-hydrophobic surfaces by imposing homogenized boundary conditions at the walls of a test channel. The boundary conditions requires the protrusion heights values to be derived from the microscopic problem.
- In order to avoid mathematical and technical difficulties, the wall texture is assumed periodic along the spanwise direction and the interface is assumed to evolve slowly in $\boldsymbol{x}$.

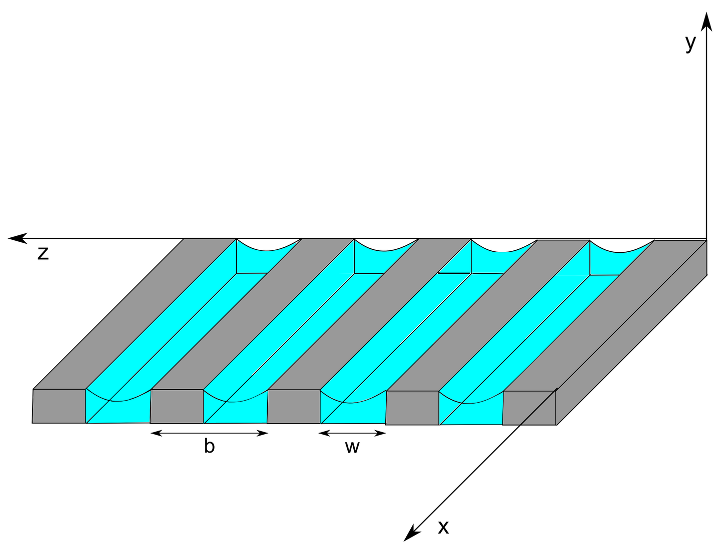

Figure 2: Typical surface corrugation studied. $\boldsymbol{b}$ is the spanwise periodicity of the wall pattern and $\boldsymbol{w}$ is the thickness of a micro-channel.

The microscopic problem

The problem is governed by the Stokes equation, which can be decoupled into two parts, called **transverse** and **longitudinal** problem

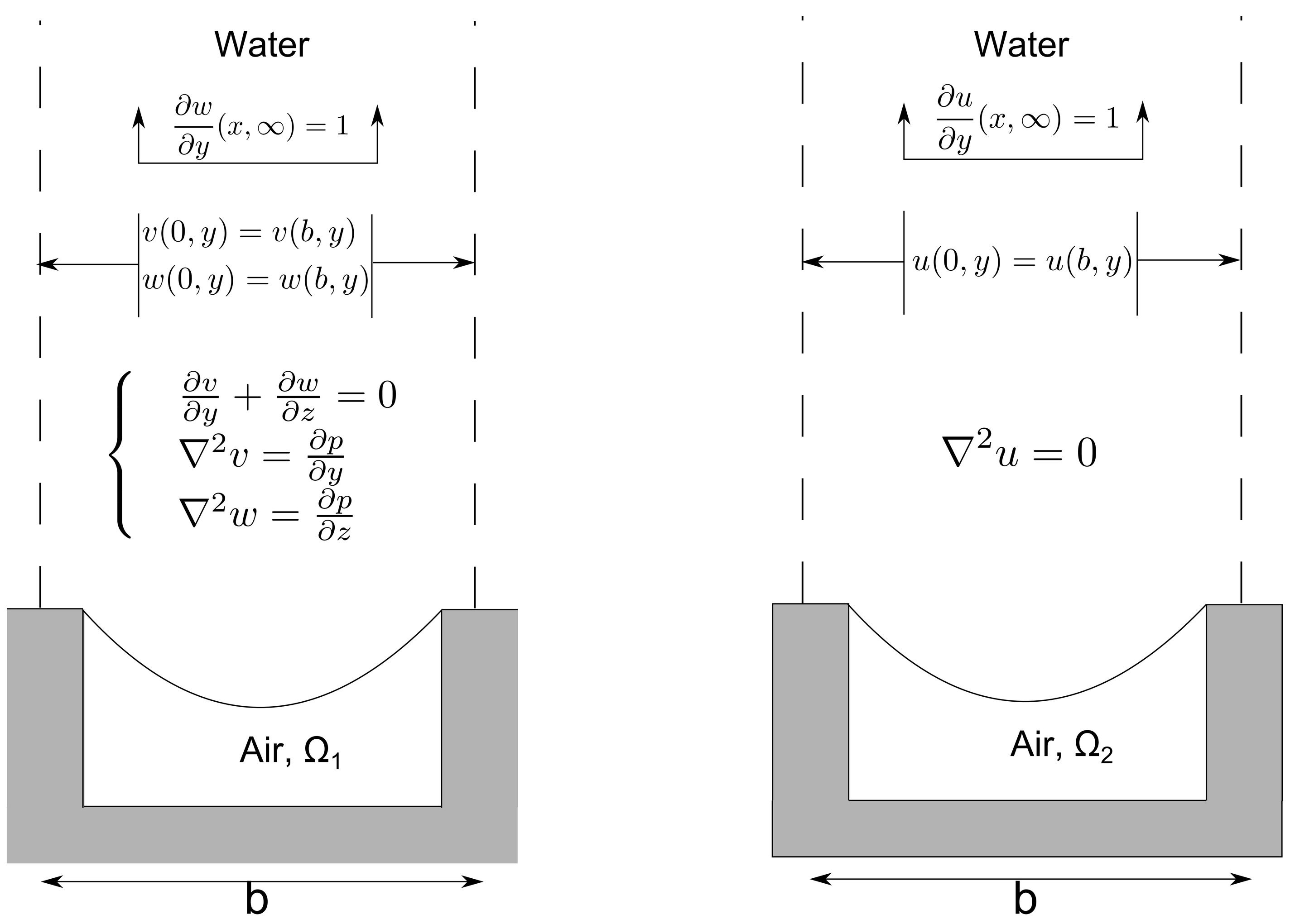

Figure 3: Geometry, governing equations and boundary conditions for the transverse (center) and the longitudinal (right) problems.

References

- J.P. Rothstein and R.J. Daniello, Superhydrophobic Surfaces for Drag Reduction, provisional U.S. patent application 61177453, filed May 12, 2009.
- W. Barthlott,C. Neinhuis, Purity of the sacred lotus, or escape from contamination in biological surfaces. *Planta*, vol 202, pp 1-8, 1997.
- C. Pozrikidis, Boundary integral and singularity methods for linearized viscous flow. *Cambridge University Press*, 1992

Boundary integral formulation

We use the boundary integral method to solve for the transverse and the longitudinal problems. The velocity field in a generic domain may be reconstructed using only the values of the velocity, $\boldsymbol{u}$, and stress fields, $\boldsymbol{f}$, on the closed boundary of the domain. This can be done introducing two integral operators, called single-layer ,$\boldsymbol{\mathcal{F}}^{SLP}$, and double-layer ,$\boldsymbol{\mathcal{F}}^{DLP}$, potentials [3]. After mathematical manipulation, the governing equations can be recast in integral form as

$$\begin{cases} \alpha u_j(x_0) = -\mathcal{F}_j^{SLP}(x_0, f; \mathsf{W}) - \mathcal{F}_j^{SLP}(x_0, f; \mathsf{T}) + \hat{\mathcal{F}}_j^{DLP}(x_0, u; \mathsf{T}) \\ -\mathcal{F}_j^{SLP}(x_0, \Delta f; \mathsf{I}) + (\lambda - 1)\hat{\mathcal{F}}_j^{DLP}(x_0, u; \mathsf{I}) \\ \\ \alpha u(x_0) = -\lambda \mathcal{F}^{SLP}(x_0, \nabla u \cdot n; \mathsf{W}_3) + \hat{\mathcal{F}}^{DLP}(x_0, u; \mathsf{T}) \\ -\mathcal{F}^{SLP}(x_0, \nabla u \cdot n; \mathsf{W}_1 + \mathsf{W}_2 + \mathsf{T}) + (\lambda - 1)\hat{\mathcal{F}}^{DLP}(x_0, u; \mathsf{I}) \end{cases}$$

with:

$$\begin{cases} \alpha = \frac{1+\lambda}{2}, \text{ if } & x_0 \in \mathsf{I}, \\ \alpha = \frac{1}{2}, \text{ if } & x_0 \in \mathsf{T} \cup \mathsf{W}_1 \cup \mathsf{W}_2, \\ \alpha = \frac{\lambda}{2}, \text{ if } & x_0 \in \mathsf{W}_3, \\ \alpha = \lambda, \text{ if } & x_0 \in \Omega_2/\{\mathsf{T}+\mathsf{I}+\mathsf{L}+\mathsf{R}+\mathsf{W}_1+\mathsf{W}_2\}, \\ \alpha = 1, \text{ if } & x_0 \in \Omega_1/\{\mathsf{W}_3+\mathsf{I}\}. \end{cases}$$

Solution in a microscopic unit cell

We calculate the value of protrusion heights for different rigidity of the fluid interface, express in term of the capillary number $\boldsymbol{Ca} = \frac{\boldsymbol{\mu u_\tau}}{\boldsymbol{\sigma}}$ and different air fractions trapped into the cavity.

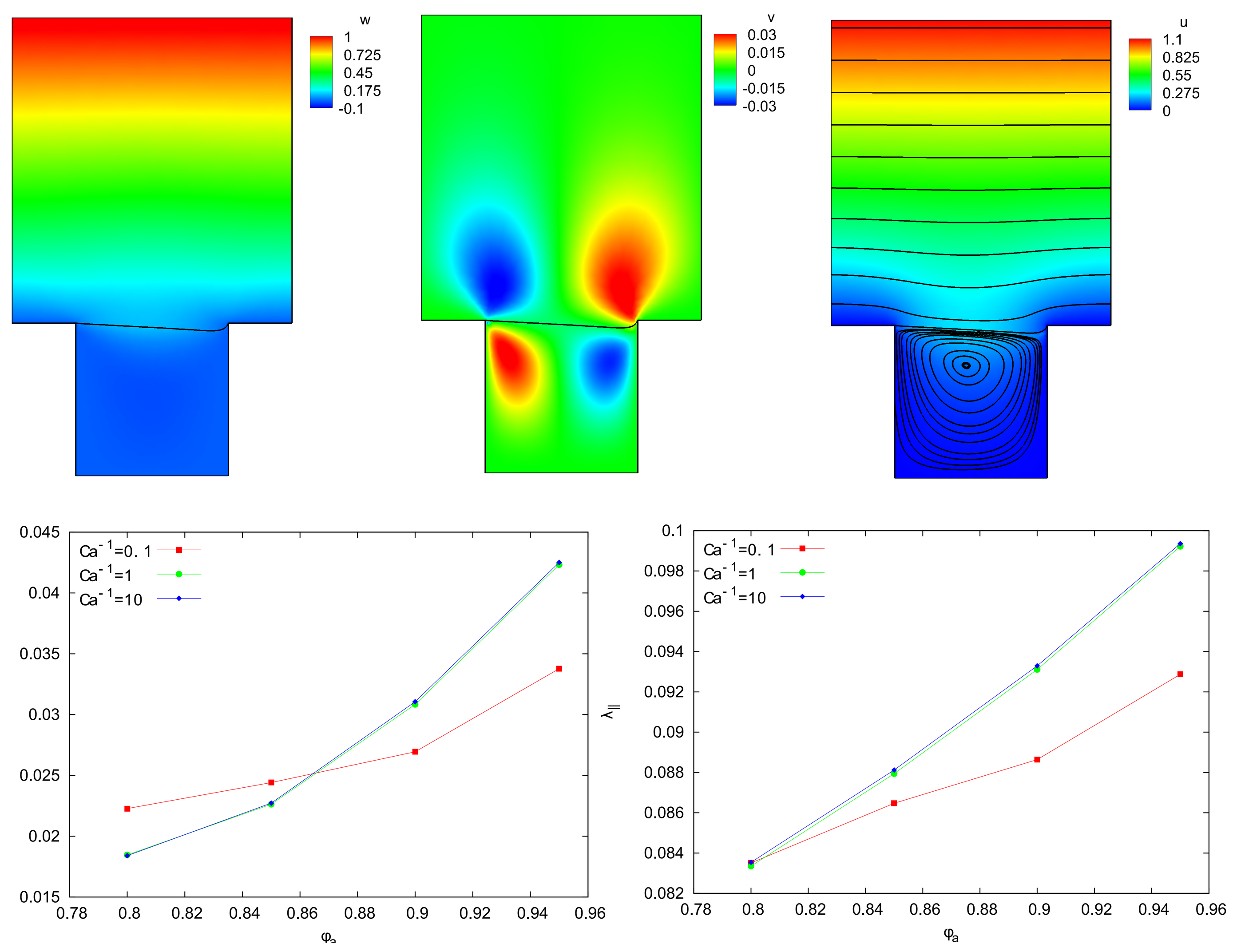

Figure 4: Computed velocity field for a flow over super-hydrophobic surfaces and associated protrusion heights for different air fractions and capillary numbers.

The macroscopic problem

Direct numerical simulations in a standard channel of dimension $\boldsymbol{6H \times 2H \times 3H}$ are employed to measure the drag reduction induced by the super-hydrophobic coating. The boundary conditions at channel lower wall reads

$$\begin{cases} u = \tilde{\lambda}_{||}\frac{\partial u}{\partial y}, & \tilde{\lambda}_{||} = \frac{b}{H}\lambda_{||} \\ v = 0, & \\ w = \tilde{\lambda}_{\perp}\frac{\partial w}{\partial y}, & \tilde{\lambda}_{\perp} = \frac{b}{H}\lambda_{\perp}. \end{cases}$$

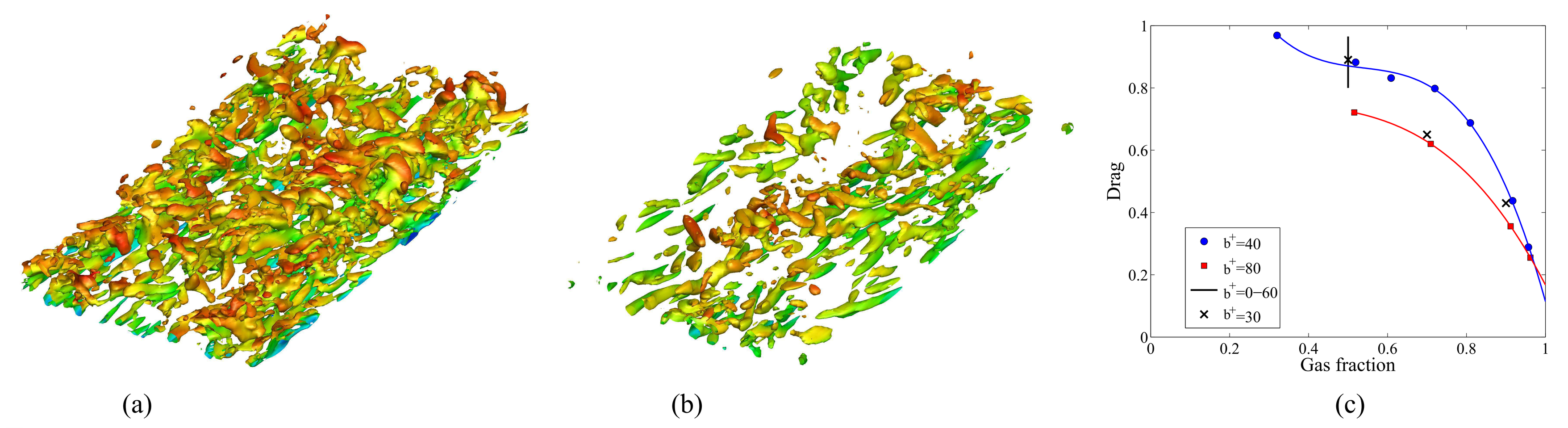

Figure 5: (a-b) Iso-surfaces of the Q-criterion colored with the streamwise velocity component; (c) Comparison between DNS and experiments by Park et al. [4]. Blue and red lines represent the least square fit of the experimental data; black solid line is the range of values achieved at $\boldsymbol{GF} = \frac{\boldsymbol{w}}{\boldsymbol{b}} = \boldsymbol{0.5}$ for different values of the spanwise texture periodicity in wall unit $\boldsymbol{b}^+$. Crosses are additional simulations at different values of GF.

Acknowledgment

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